Morphology of Standing Oblique Detonation Waves

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Proposals have been made to utilize stabilized oblique detonation waves (ODWs) for the propulsion of hypersonic air-breathing vehicles and hypervelocity mass launchers. There exists hypersonic flight regimes where premixing of fuel and air may be desirable or unavoidable due to finite chemical induction times. Consequently, it is essential to understand under what conditions detonations may occur in order to design supersonic combustors to either avoid or utilize them efficiently. A theoretical analysis is made of supersonic flow of a combustible gas mixture past a wedge or an inclined wall, which shows that, for approach velocities roughly 25% or more greater than the Chapman-Jouguet velocity of the reactant mixture, there exists a usefully wide range of turning angles within which ODWs may be attached or stabilized. For smaller wedge angles, either an incomplete ODW, shock-induced combustion, or no combustion at all may ensue. For larger wedge angles, the wave will detach and form an overdriven normal detonation or normal shock-induced combustion wave immediately upstream of the stagnation point, decaying off axis to either a single oblique Chapman-Jouguet wave, or bifurcating to form an oblique shock followed by a shock-induced deflagration.

I. Introduction

THE use of a stabilized normal detonation wave (NDW) for hypersonic aircraft propulsion was proposed in 1946 by Roy.[1] Propulsion cycle analyses for NDW engines[2-4] demonstrated thermodynamic feasibility, but the difficulty of stabilizing an NDW in steady flow[5-8] appeared to necessitate variable engine geometry for propulsion applications. In addition, since the flow downstream of a normal detonation is necessarily sonic or subsonic, very high static temperatures would result with increasing flight Mach numbers, causing severe dissociation of combustion products. As a result, the NDW cycle performance falls off sharply with flight Mach numbers above 6.[2]

In 1958, Ferri[9] proposed the use of deflagration or diffusive combustion in order to maintain supersonic flow throughout the combustor. The theoretical cycle performance of a supersonic combustion ramjet (SCRJ)[10,11] permitted flight Mach numbers significantly greater than an NDW cycle, so that the NDW ramjet was effectively ruled out for hypersonic aircraft propulsion.[12]

Dunlap et al.[2] first suggested the use of a stabilized oblique detonation wave (ODW) for hypersonic ramjet propulsion. Since only the normal component of flow leaving an ODW is sonic or subsonic, supersonic flow can be maintained throughout the combustor, as with the SCRJ. Assuming that the ODW could be stabilized by wave attachment to a simple wedge, the variable combustor geometry requirement would be eliminated. The ODW engine would thereby retain the two principal advantages of the NDW engine over the SCRJ: very short combustor length and less inlet diffusion.[2] The major drawback of the ODW engine foreseen in Ref. 2 was the requirement to premix the fuel and air while avoiding ignition upstream of the stabilizing wedge. However, premixing was later shown to offer significant benefits to SCRJ cycle performance as well,[13] and a recent finite-rate chemistry analysis[14] suggests that premixing may be necessary for SCRJ combustion in the flight Mach number range 8-16.

Experimental attempts to stabilize oblique detonation waves were reported in the late 1950s and early 1960s.[6,15,16] Unfortunately, these experiments were limited to low approach Mach numbers and, as the present paper demonstrates, such limited heat release (substoichiometric) as to effectively guarantee inconclusive results. Consequently, reported experimental results were very controversial and their interpretation was clouded by semantic arguments over the definition of the term "detonation."[17-21] The final blow to ODW research was the post-Sputnik shift of national aerospace propulsion research priorities toward chemical rockets, which effectively halted all research related to air-breathing hypersonic propulsion. As a result, questions concerning flameholding, stability, and the degree of thermodynamic irreversibility (total pressure loss) of an ODW are still unresolved.[22]

With the recent revival of interest in hypersonic air-breathing propulsion, interest has also been renewed in the ODW engine as an alternative to the SCRJ engine to propel various proposed hypersonic air-breathing vehicles for transcontinental or single-stage-to-orbit flights.[23-25] Hertzberg et al.[26] have recently proposed the use of stabilized detonation waves, both normal and oblique, among many alternative combustion modes for a "ram accelerator," a device in which a shaped projectile is accelerated by combustion while transiting a tube filled with a reactive gaseous mixture.

Detailed analysis procedures have been developed in support of installed performance predictions for ODW engines[27] and ODW-driven ram accelerators[26] based on the assumption that an ODW can in fact be stabilized at specific, mission-required conditions of pressure, temperature, approach flow velocity, fuel type, and fuel-air ratio—an assumption that, it is emphasized, is supported only by controversial theoretical

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analysis and for which there is little supporting experimental evidence. Consequently, it is the objective of the present paper to clarify the interactions between thermodynamic, chemical-kinetic, and gasdynamic considerations that determine the required envelope of operating parameters within which a stable ODW with acceptable total pressure loss can be realized, and to critically review the existing literature in light of this understanding.

II. Terminology

Attention will be devoted exclusively to combustion processes in homogeneous reactive mixtures of ideal gases. We begin with Strehlow's definition of a detonation[8]: "At the present time the term *detonation* should be applied only to processes where a shock-induced combustion wave is propagating through a reactive mixture or pure exothermic compound." Strehlow's definition is in the context of free-running or self-propagating detonation waves, in which the energy to support the wave structure comes entirely from the exothermic combustion reaction. For detonation waves stabilized in steady flows, however, where most of the stabilizing energy comes from the kinetic energy of the approach stream, a more refined definition is required: When a shock-induced combustion wave follows so closely on the igniting shock wave that the two waves are fully pressure coupled, the resulting shock/combustion wave structure is a detonation. By contrast, a shock-induced combustion wave results whenever the igniting shock wave is uncoupled, clearly nonreactive, and followed (typically after a measurable induction or ignition delay period) by a distinct, temporally or spatially resolvable deflagration wave.

This terminology is consistent with that of Nicholls[17]: "Furthermore, with no strong interaction between shock and combustion, it hardly seems plausible that these could rightfully be called detonations. In fact, we have preferred to consider such phenomena as shock ignition." It is also consistent with that of Robins and Cunningham[16]: "Shock-induced combustion differs from detonation in gases in that the chemical reaction behind the shock does not necessarily affect the shock, but it is similar in that the reaction is initiated by the shock wave, which heats the gas to a reacting temperature."

Detonation waves will be further classified as overdriven, Chapman-Jouguet (C-J), or underdriven, depending on whether the normal component of flow velocity following the wave is subsonic, sonic, or supersonic, respectively.[8] When the supersonic approach stream velocity exceeds the C-J velocity for the reactant mixture, the velocity will be called superdetonative.[26] As with nonreactive shock waves, an ODW can be attached or detached to or from the stabilizing wedge or wall protrusion and can be either strong or weak, depending on whether the flow velocity immediately following the wave is subsonic or supersonic, respectively. Finally, we admit the possibility of incomplete detonations, in which combustion is not completed within the coupled wave structure.[28]

These definitions are consistent with the one-dimensional Zeldovich-von Neumann-Doring (ZND) model of detonation wave structure,[8] in which the end states are described by the jump conditions for a planar detonation wave, but the upstream plane of the wave is a nonreactive shock with a "von Neuman spike" overpressure, followed by relaxation to the downstream end state through a weak (subsonic-to-subsonic) deflagration.

III. Analysis

Much of the analysis to follow has appeared before in various forms,[29-36] but nowhere in the open literature in the degree of detail, or with the appropriate terminology, to support the arguments offered and conclusions drawn in the present paper.

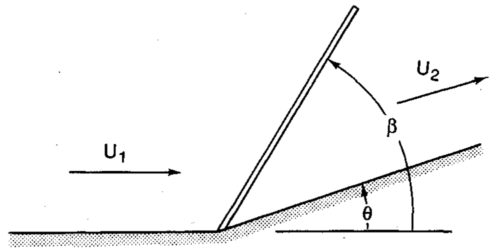

Fig. 1 Plane oblique shock geometry.

Figure 1 depicts the supersonic flow of an ideal gas past a wedge with an inclination angle θ. Because of the presence of the wedge, an attached planar oblique shock wave is formed at an angle β from the initial direction of flow. Of interest are the possible changes in the kinematic and thermodynamic state of the gas in case sensible heat is released by exothermic chemical reaction occurring within the shock wave. This heat release model is consistent with our definition of a detonation and permits the oblique wave to be treated as a discontinuity.

For adiabatic or nonadiabatic flows, with or without chemical change, the governing equations are

Conservation of mass:

$$\rho_1 u_{1n} = \rho_2 u_{2n} \tag{1}$$

Conservation of normal momentum:

$$p_1 + \rho_1 u_{1n}^2 = p_2 + \rho_2 u_{2n}^2 \tag{2}$$

Conservation of tangential momentum:

$$(\rho_1 u_{1n}) u_{1t} = (\rho_2 u_{2n}) u_{2t} \tag{3}$$

or

$$u_{1t} = u_{2t} = u_t \tag{4}$$

Conservation of energy:

$$h_1 + (u_{1n}^2/2) = h_2 + (u_{2n}^2/2) \qquad (\text{since } u_{1t}^2 = u_{2t}^2) \tag{5}$$

In Eq. (5), the enthalpies h_1 and h_2 include both the sensible enthalpy and the chemical enthalpy (enthalpy of formation).

Accurate analysis of performance at high temperature and pressure requires the use of variable specific heats and complex chemical equilibrium and kinetics calculations,[37] assumptions that do not lend themselves to the expository analysis presented here. Therefore, for purposes of exposition, further simplifying assumptions will be made. Assuming there is no chemical change but there is the addition of Q units of heat per unit mass to represent the sensible heat release due to combustion, Eq. (5) may be rewritten as

$$h_1 + Q + (u_{1n}^2/2) = h_2 + (u_{2n}^2/2) \tag{6}$$

With the further assumption of constant specific heat capacities, Eq. (6) becomes

$$c_p T_1 + Q + (u_{1n}^2/2) = c_p T_2 + (u_{2n}^2/2) \tag{7}$$

In addition to the four conservation equations, Eqs. (1-7), the fixed-composition, ideal gas equation of state is assumed to apply,

$$p = \rho RT \tag{8}$$

The usual kinematic relations for oblique shocks are given by

$$u_{1n} = u_1 \sin\beta, \qquad u_{2n} = u_2 \sin(\beta - \theta) \tag{9}$$

$$u_{1t} = u_1 \cos\beta, \qquad u_{2t} = u_2 \cos(\beta - \theta) \tag{10}$$

As shown by Gross,[33] Eqs. (1), (4), (9), and (10) may be combined to give the important working relations

$$X \equiv \frac{\rho_1}{\rho_2} = \frac{u_{2n}}{u_{1n}} = \frac{\tan(\beta - \theta)}{\tan\beta} \tag{11}$$

The density ratio defined by Eq. (11) may be substituted into the n-momentum equation, Eq. (2), to give

$$p_2 = p_1 + \rho_1 u_{1n}^2 (1 - X) \tag{12}$$

and into either energy equation, Eq. (5) or (7), to give

$$h_2 = h_1 + (u_{1n}^2/2)(1 - X^2) \tag{13}$$

or

$$\frac{T_2}{T_1} = 1 + \frac{u_{1n}^2}{2\,c_p T_1}(1 - X^2) + \frac{Q}{c_p T_1} \tag{14}$$

Focusing on the conservation equations for nonreacting, ideal gases with constant specific heats and prescribed heat addition Q and introducing the Mach number

$$M \equiv \frac{u}{\sqrt{kRt}} \tag{15}$$

where $k \equiv C_p/C_v$ is the specific heat ratio, the n-momentum equation, Eq. (12), may be rewritten as

$$p_2/p_1 = 1 + k\,M_1^2 \sin^2\beta\,(1 - X) \tag{16}$$

and the energy equation, Eq. (14), may be rewritten as

$$T_2/T_1 = 1 + \tilde{Q} + [(k-1)/2]\,M_1^2 \sin^2\beta\,(1 - X^2) \tag{17}$$

where $\tilde{Q}$ is the second Damkohler parameter,

$$\tilde{Q} \equiv Q/c_p T_1 \tag{18}$$

Recalling the definition of Eq. (11), $X \equiv \rho_1/\rho_2$, the gas equation of state, Eq. (8), may be used, together with Eqs. (16) and (17), to eliminate the property ratios T_2/T_1 and P_2/P_1, resulting in

$$\tilde{Q} = -[(k+1)/2]X^2 M_1^2 \sin^2\beta + (1 + k\,M_1^2 \sin^2\beta)\,X - (1 + [(k-1)/2]\,M_1^2 \sin^2\beta) \tag{19}$$

Equation (19) may be rewritten as a quadratic algebraic equation in the density ratio X, which may be solved for the two roots (with new notation $M_{1n} \equiv M_1 \sin\beta$)

$$X = \frac{1 + k\,M_{1n}^2 \pm \sqrt{(M_{1n}^2 - 1)^2 - 2\,(k+1)\,M_{1n}^2\,\tilde{Q}}}{(k+1)\,M_{1n}^2} \tag{20}$$

To understand Eq. (20), consider first the adiabatic case, for which $\tilde{Q} = 0$. In this case, Eq. (20) simplifies to ($\tilde{Q} = 0$):

$$X = \frac{1 + k\,M_{1n}^2 \pm (M_{1n}^2 - 1)}{(k+1)\,M_{1n}^2} \tag{21}$$

which may be further simplified to give the two roots ($\tilde{Q} = 0$):

$$X^+ = 1, \qquad X^- = \frac{2 + (k-1)\,M_{1n}^2}{(k+1)\,M_{1n}^2} \tag{22}$$

which are the familiar trivial ($X = 1$) and standard results for oblique shock waves.[38,39] The single nontrivial solution (for $\tilde{Q} = 0$) is represented in Fig. 2, from which it can be seen that, for any given Mach number M_1 and wall angle θ, there are two wave angles β that satisfy the conservation equations, corresponding to weak (smaller β) and strong (larger β) oblique shock waves.

For the case of $\tilde{Q} > 0$, both roots of Eq. (20) are nontrivial. The nature of the two solutions of Eq. (20), also represented in Fig. 2, can be analyzed by first considering the limiting case when a single root exists, for which case the discriminant of Eq. (20) is equal to zero:

$$D \equiv (M_{1n}^2 - 1)^2 - 2\,(k+1)\,M_{1n}^2\,\tilde{Q} = 0 \tag{23}$$

and the corresponding value of X is

$$X_{D=0} = \frac{1 + k\,M_{1n}^2}{(k+1)\,M_{1n}^2} \tag{24}$$

By substitution of the zero-discriminant expression for X from Eq. (24) into the conservation and state equations, it can be shown that the normal component of Mach number downstream of the wave is equal to unity.[33] Thus, the two roots of Eq. (20) can be classified by whether M_{2n} is less than or greater than unity.

To be consistent with the recommended terminology and to avoid confusion with the terms weak and strong as applied to oblique shock waves (the lower and upper branches, respectively, of the $\tilde{Q} = 0$ curve of Fig. 2), the following nomenclature will be adopted for subsequent analysis and discussion:

Weak oblique shock waves with $\tilde{Q} > 0$ for which $M_{2n} > 1$ will be called weak underdriven ODWs.

Weak oblique shock waves with $\tilde{Q} > 0$ for which $M_{2n} < 1$ will be called weak overdriven ODWs.

Strong oblique shock waves with $\tilde{Q} > 0$ are all overdriven ($M_{2n} < 1$), and so, to avoid redundancy, they will be called simply strong ODWs.

It is thus seen that there are three distinct classes of attached ODWs that must be considered in subsequent discussions of

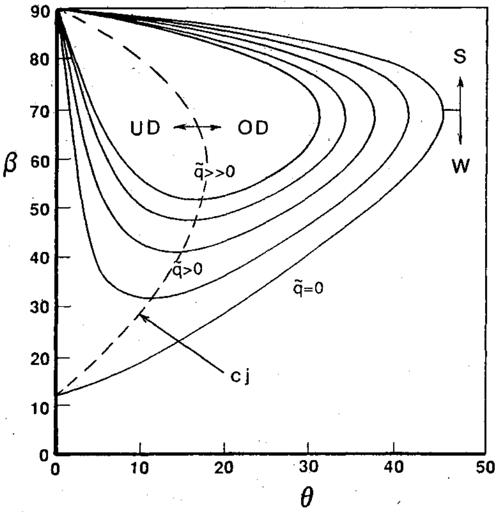

Fig. 2 Variation of oblique shock wave angle with turning angle for variable specific heat addition q and constant freestream Mach number and temperature: UD = underdriven regime, OD = overdriven regime, S = strong shock, W = weak shock, cj = locus of Chapman-Jouguet states.

detonation wave stability for application to propulsion devices.

Returning to analysis of the two branches of Eq. (20), the branch with the larger value of X corresponds to the underdriven or left branch of the constant $\tilde{Q} > 0$ curves of Fig. 2, whereas the smaller valued X solution corresponds to the overdriven or right branch. The boundary between the two branches is delimited by the C-J state, which is the point of minimum wave angle β on each locus of states for given $\tilde{Q} > 0$ in Fig. 2. The C-J state is of critical significance for the case of heat release due to homogeneous chemical reaction in gases because underdriven weak waves cannot occur for reasons that will be given in a subsequent section. As a consequence, it can be seen by inspection of Fig. 2 that, for any given Mach number M_1, specific heat release Q, and initial static temperature T_1, the C-J point represents the minimum turning angle θ for which stable heat release Q can occur. However, it is emphasized that although $M_{2n} < 1$ is a necessary condition for an ODW to attach or stabilize, it is not sufficient. The present analysis describes the flow behavior only when finite-rate chemistry permits detonation to occur for C-J and overdriven ODWs.

It is also apparent from inspection of Fig. 2 that, for any given M_1 and $\tilde{Q}$, there is a maximum turning angle θ_{max} beyond which an ODW will detach, just as with nonreactive oblique shocks, except that in this case an overdriven normal detonation or a normal shock-induced combustion wave will form just upstream of the junction of wedge or ramp with the wall, rather than a normal shock. The effect of increasing amounts of heat release is therefore seen to reduce the range of permitted turning angles for attached oblique waves.

Having established the significance of the C-J state as a lower bound on wedge angles for possibly stable oblique detonations, it is again necessary to introduce additional notation and definitions in order to continue the analysis.

Note that Eq. (23) can be solved explicitly for $\tilde{Q}$,

$$\tilde{Q} = \frac{(M_{1n}^2 - 1)^2}{2\,(k+1)\,M_{1n}^2} \tag{25}$$

Since Eq. (25) is valid only for the C-J condition ($M_{2n} = 1$), then for the special case of $\beta = 90$ deg, which is a normal detonation, we define the C-J heat addition,

$$\tilde{Q}_{cj} \equiv \frac{(M_1^2 - 1)^2}{2\,(k+1)\,M_1^2} \tag{26}$$

which represents the maximum heat addition possible for a given value of approach Mach number M_1. If this amount of heat were to be released, a normal C-J detonation wave would stand in the flow, which would cause it to be thermally choked with $M_2 = 1$. From Eq. (26), it can be seen that $\tilde{Q}_{cj}$ varies approximately with the square of the approach Mach number. Therefore, $\tilde{Q}/\tilde{Q}_{cj}$ is a measure scaled on {0,1}, which indicates how close any given amount of heat release comes to choking the flow. From Fig. 2, it is clear that the smallest amount of heat release possible provides the least thermal occlusion to the flow.

Equation (23) may also be solved explicitly for M_{1n}:

$$M_{1n}^2 = 1 + \tilde{Q}(k+1) \pm \sqrt{[1 + \tilde{Q}(k+1)]^2 - 1} \tag{27}$$

For $\tilde{Q} > 0$, the (+) root of Eq. (27) is applicable. The (−) root corresponds to a C-J deflagration ($M_1 < 1$), which is not meaningful for a supersonic approach flow.[8,38,39]

When $\beta = 90$ deg, the C-J Mach number may be defined from Eq. (27),

$$M_{cj}^2 \equiv [1 + \tilde{Q}(k+1)] \pm \sqrt{[1 + \tilde{Q}(k+1)]^2 - 1} \tag{28}$$

Although the principal motivation for Eq. (28) is merely convenience of notation, M_{cj} may be interpreted as the Mach number of a free-running normal detonation wave in which $\tilde{Q}$ of heat is released in an initially quiescent mixture at T_1.[8,40]

Finally, a C-J density ratio, X_{cj} also may be defined from Eq. (24). Using Eq. (27) and the notation of Eq. (28),

$$X_{cj} \equiv \frac{1 + k\,M_{cj}^2}{(k+1)\,M_{cj}^2} \tag{29}$$

The C-J wave angle β_{cj} is given by

$$\beta_{cj} = \sin^{-1}(M_{cj}/M_1) \tag{30}$$

From Eqs. (11) and (24) and the trigonometric identity

$$\tan(\sin^{-1}\beta) = \frac{\beta}{\sqrt{1-\beta)}} \tag{31}$$

the C-J turning angle θ_{cj} can be shown to be

$$\theta_{cj} = \beta_{cj} - \tan^{-1}\left[\frac{1 + k\,M_{cj}^2}{(k+1)\,M_{cj}^2\,\sqrt{(M_1/M_{cj})^2 - 1}}\right] \tag{32}$$

Since the C-J state corresponds to a limiting condition for stabilizing an ODW, further analysis is of interest. Figure 3 illustrates how θ_{cj} of Eq. (32) varies with M_1 for a given reactant mixture (T_1, k, Q) for which the C-J Mach number is uniquely determined by Eq. (28). The maximum value of θ_{cj} occurs at a Mach number

$$M_1\Big|_{max\,\theta_{cj}} = \frac{1 + (2k+1)\,M_{cj}^2}{(k+1)} \tag{33}$$

Figure 3 also shows the variation with Mach number of the maximum turning angle θ_{max}, corresponding to wave detachment from the wedge.

Figure 3 clearly illustrates the inherently superdetonative character of stable ODWs: at approach velocities below the C-J Mach number for the reactive mixture, no stable ODW is possible for the prescribed heat release Q. Equation (33) shows that superdetonative approach velocities—roughly 25% in excess of the C-J velocity—are required for the existence of a useful range of turning angles.

Figures 4–7 illustrate the same relationship between wall angle, wave angle, and heat release, as illustrated in Fig. 2, except that the calculations have been performed for variable specific heats and for chemical equilibrium of the burned gas, and the heat release parameter is represented by the fuel/air equivalence ratio ϕ rather than $\tilde{Q}$. Figures 4–6 show that, for a given approach pressure and Mach number, the effect of increasing inlet temperature is to increase the capacity of the ODW to accept full stoichiometric heat release. The effect of increasing Mach number at the same pressure and temperature can be clearly seen by comparison of Figs. 4 and 7.

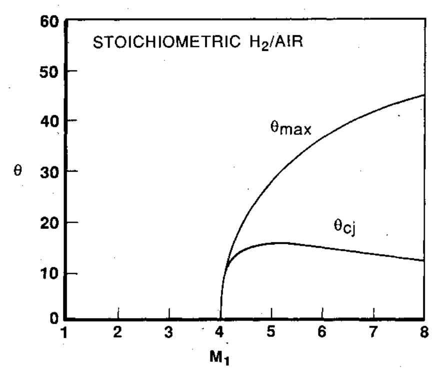

Fig. 3 Variation of maximum and minimum turning angles with freestream Mach number for stable oblique detonation waves for stoichiometric hydrogen/air at 555 K.

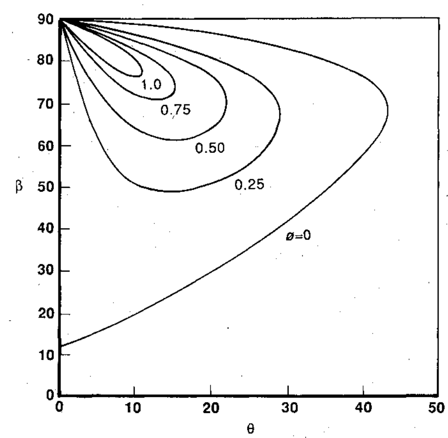

Fig. 4 Wave angle vs turning angle for Mach 5, $T = 278$ K, $P = 0.1$ atm.

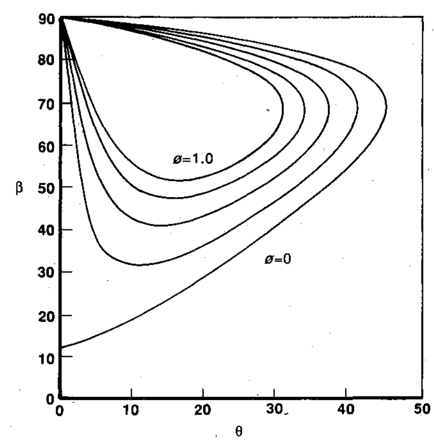

Fig. 5 Wave angle vs turning angle for Mach 5, $T = 555$ K, $P = 0.1$ atm.

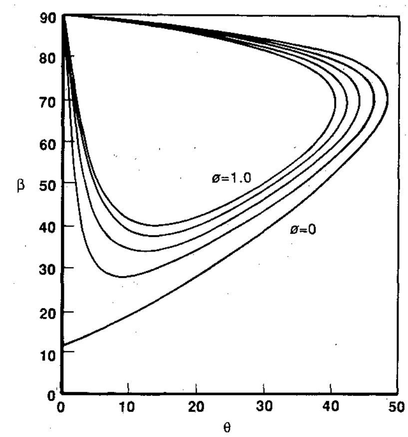

Fig. 6 Wave angle vs turning angle for Mach 5, $T = 833$ K, $P = 0.1$ atm.

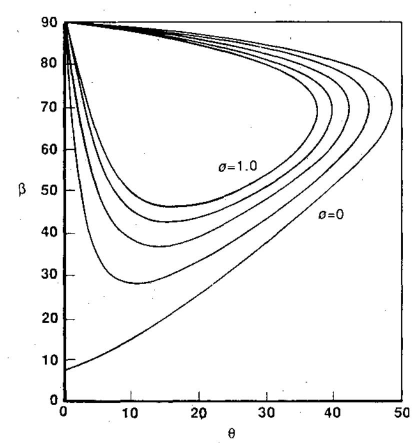

Fig. 7 Wave angle vs turning angle for Mach 8, $T = 278$ K, $P = 0.1$ atm.

The total temperature ratio, total pressure ratio, and entropy change for constant specific heats may be expressed by standard gasdynamic relations.[38,39] Loss of total pressure results from both heat release and shock irreversibility. Since total pressure loss varies monotonically with oblique shock wave angle,[38,39] the C-J turning angle corresponds to the minimum total pressure loss for given approach conditions. In addition, in the vicinity of the C-J state, the wave angle and, therefore, the total pressure loss is insensitive to the turning angle θ, within the limits illustrated in Fig. 3. Consequently, fixed wall geometry allowing for varying degrees of detonation overdrive near the C-J point could possibly be utilized in ODW engine design. Finally, as the stoichiometric heat release $\tilde{Q}_{\text{stoich}}$ is fixed independently of flight Mach number and $\tilde{Q}_{\text{cj}}$ increases with increasing flight Mach number, both $\tilde{Q}/\tilde{Q}_{\text{cj}}$ and θ_{cj} decrease monotonically with flight Mach number, so that the theoretical ODW total pressure loss approaches that of the SCRJ engine at increasing flight Mach numbers. This fact, together with simpler engine geometry with concomitant increases in component efficiencies, results in possible superior predicted performance of the ODW engine compared to the SCRJ engine at flight Mach numbers beyond 12.[27]

IV. Flameholding and Stability

In the terminology of continuous combustion devices, stability refers to flameholding, for example by convective recirculation in the wake of a gutter in a subsonic combustion ramjet or turbojet afterburner. In an ideal SCRJ, the static temperature of the air is assumed to be raised sufficiently due to ram compression that the fuel and air react spontaneously upon mixing, so that the resulting flame holds at the mixing interface without the need for convective recirculation.

Under conditions when two supersonic streams of fuel and air can be mixed without immediate chemical reaction occurring, an abrupt change in wall angle or a protrusion into the stream would cause an oblique shock wave (or system of oblique shocks) through which the reactant mixture would pass. If the reactant static pressure and temperature were raised sufficiently by the oblique shock(s), either shock-induced combustion or a complete or incomplete ODW might result. Although it is well known that nonreacting oblique shocks are hydrodynamically stable,[38,39] it is necessary to explore under what conditions ODWs may be similarly stable.

As the flow is turned, the inclined wall (wedge) provides the pressure support for the attached oblique wave, just as a piston provides the supporting pressure for an overdriven normal detonation wave.[8] At superdetonative velocities, the predominant source of mechanical energy to support an ODW thus comes from the freestream, rather than from the work of volume expansion due to chemical heat release, as is the case for self-propagating normal detonation waves.

As the approach Mach number is increased, the ratio of energy supplied by exothermic chemical reaction to that supplied by the freestream decreases, because the heat released by combustion is fixed by the stoichiometric mixture ratio. Consequently, measured fuel-air detonation limits for self-propagating detonation waves are not applicable to stabilized ODWs.

The limits of hydrodynamic stability of an ODW can be established by straightforward reasoning based on stable wave mechanics.[30] From Eqs. (9) and (15)

$$M_{2n} = M_2 \sin(\beta - \theta) \tag{34}$$

which may be solved explicitly for the wave angle β,

$$\beta = \theta + \sin^{-1}(M_{2n}/M_2) \tag{35}$$

It may be seen from Eq. (35) that, for the C-J condition $M_{2n} = 1$, the resulting wave angle β_{cj} is exactly parallel to the post-ODW Mach wave or left-running characteristic[38,39] emanating from the inclined wall (wedge) downstream of the wave.

For overdriven ODWs, for which $M_{2n} < 1$, it can be seen from Eq. (35) that downstream Mach waves intersect the oblique shock wave: $\beta < [\theta + \sin^{-1}(1/M_2)]$. Thus, overdriven ODWs are stable for any amount of heat release, as is the case for nonreacting attached oblique shocks, for which M_{2n} is always less than unity.

In the case of underdriven waves, for which $M_{2n} > 1$, Eq. (35) shows that downstream Mach waves diverge from the oblique shock wave: $\beta > [\theta + \sin^{-1}(1/M_2)]$. Consequently, since the presence of the wall cannot be transmitted to an underdriven oblique wave, such a wave cannot be attached to and supported by a wedge.

Figure 8 illustrates how the ODW adjusts itself in case the flow turning angle is less than the minimum turning angle θ_{cj} for a given stoichiometry and complete combustion. In Fig. 8, the wedge angle θ_1 is initially equal to the limiting turning angle θ_{cj} given by Eq. (32) for a specified amount of heat q_1 released within the oblique wave of angle β_1. If the wedge angle were to be reduced to θ_2, the conservation laws for fixed $q = q_1$ would require an underdriven wave to result, corresponding to point 2 in Fig. 8. However, as point 2 is not stable and the flow is still required to turn through the angle θ_2, the only stable endpoints possible are those lying between the limits of point 2′, no detonation (that is, at most shock-induced combustion), and incomplete ODW's with partial heat release at most q_2, point 2″, determined by the condition $\theta_{cj} = \theta_2$.

If the wedge angle is sufficiently great that the ODW detaches, or in the case of a blunt body immersed in the flow or protruding from the wall as illustrated in Figs. 9 and 10, either normal shock-induced combustion or a normal detonation wave will occur immediately upstream of the object, with maximum (normal shock or detonation) strength along the stagnation streamline. Off axis, the normal wave will decay into an oblique wave that weakens with increasing distance from the body and that may either bifurcate into an oblique shock-induced deflagration wave or decay into an oblique C-J wave.[35,41]

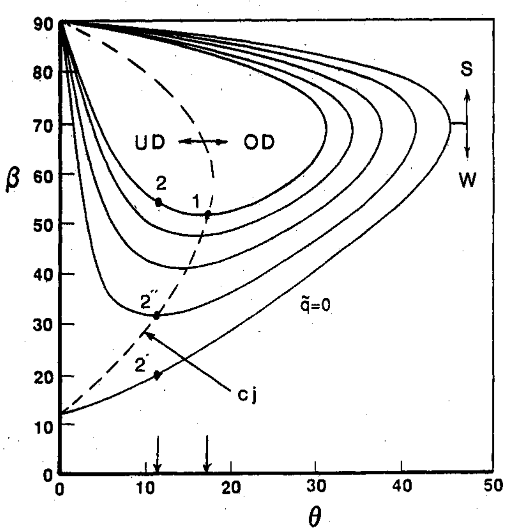

Fig. 8 Wave angle vs turning angle, illustrating case where turning angle is less than C-J turning angle, leading to incomplete detonation: UD = underdriven regime, OD = overdriven regimé, S = strong shock, W = weak shock, cj = locus of Chapman-Jouguet state.

Ballistic range experiments, in which blunt- or sharp-nosed projectiles were fired into combustible gaseous mixtures,[42–44] showed that for supersonic but subdetonative projectile velocities, a periodic wave structure occurs around and in the wake of the projectile, with periodicity directly related to the induction or ignition delay time of the mixture. For superdetonative projectile velocities, the periodicity increases in frequency and decreases in strength, essentially disappearing at sufficiently high superdetonative approach velocities. However, recent theoretical analysis suggests that post bow wave, periodic detonative structures may arise even for superdetonative approach velocities if the transverse body dimension exceeds some critical multiple of the induction distance.[45]

V. Discussion

With the benefit of the preceding terminology and analysis, it is now possible to review critically the somewhat limited literature on stabilized oblique detonation waves.

Gross et al.[15,18,46] widely published schlieren photographs (Figs. 11, 4, and 10 of Refs. 15, 18, and 46, respectively) of what was identified as an ODW, stabilized on a 30-deg wedge in an 833 K total temperature, Mach 3-plus flow of hydrogen and air at stoichiometry corresponding to an effective $\tilde{Q} \approx 1.3$. Assuming $k = 1.4$, Eq. (26) predicts thermal choking at $\tilde{Q}_{cj} \approx 1.5$, so that $\tilde{Q}/\tilde{Q}_{cj} \approx 0.8$. From the present analysis, as well as that of Ref. 32 cited by Gross et al., it is apparent that the flow was very near to thermal choking and that no solutions to the governing equations exist anywhere near the stated conditions. From the wave angle of approximately 75 deg, which can be measured from the schlieren photographs, it appears likely that shock-induced combustion occurred downstream of the test section, off camera, which caused choking due to the combined thermal occlusion from heat release and flow area occlusion due to the 30-deg wedge. The combined choking caused an increased in back pressure, which in turn forced a nonreacting, strong oblique shock wave to be stabilized on the wedge. A similar interpretation of Gross' results was offered by Fletcher.[19]

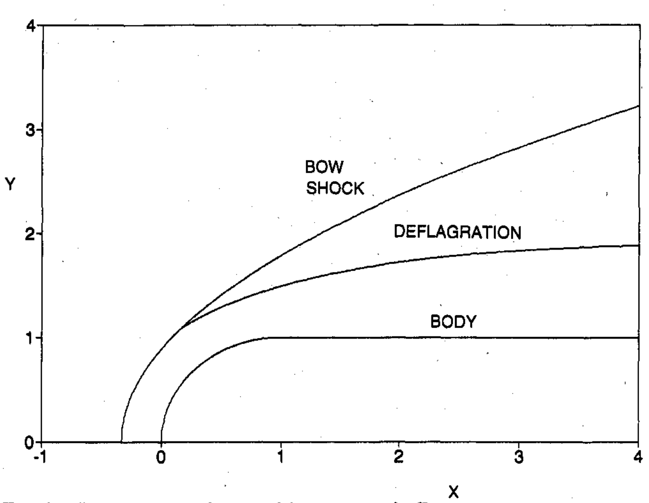

Fig. 9 Wave pattern in reacting supersonic flow past a sphere-cylinder combination at Mach 5.5, 300 K, 1 atm.[41]

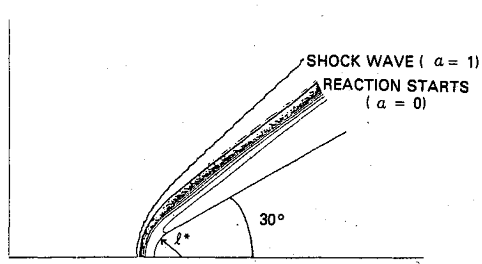

Fig. 10 Wave pattern in reacting supersonic flow past a blunted wedge at Mach 8.[45]

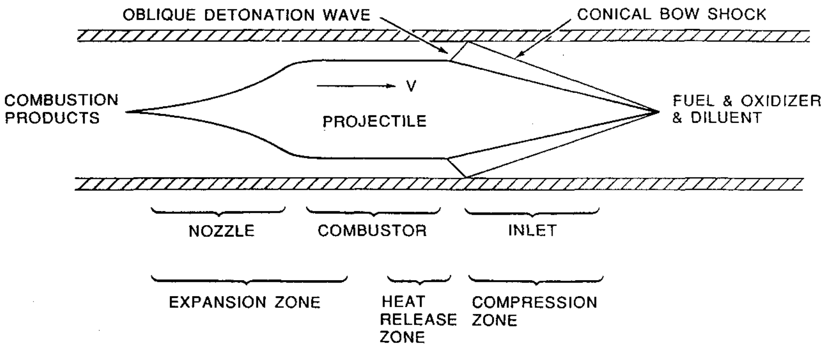

Fig. 11 Schematic of ODW-driven ram accelerator. Tube wall acts as the outer cowling of a ramjet engine as the projectile/centerbody transits the combustible mixture in a long tube.[26]

Nicholls[17] and Rutkowski and Nicholls[31] ruled out the possibility of underdriven attached weak ODWs "on the basis of entropy considerations," without further elaboration. However, a clear-cut second law argument is possible only if a ZND structure for the wave is assumed, which requires a disallowed strong deflagration (subsonic-to-supersonic heat addition without area change) following the leading von Neumann spike of the nonreacting shock.[47,48]

Rutkowski and Nicholls[31] also explored possible stable states that might result from off-C-J conditions. For wedge angles greater than θ_{cj}, they predicted simply overdriven ODWs, in agreement with the present analysis. However, for wedge angles in the underdriven regime, $\theta_{wedge} < \theta_{cj}$, they argued that since θ_{cj} is the smallest angle through which the flow can be turned for a given heat release Q, a C-J ODW would occur anyway; that is, point 1 of Fig. 8 would be realized immediately downstream of the oblique wave at angle β_1, following which a Prandtl-Meyer expansion would turn the flow isentropically back to the wedge angle θ_2. Woolard[34] and Chernyi[35] both agreed with this predicted behavior for off-C-J conditions. Apparently, none of these three investigators[31,34,35] considered the possibility of incomplete detonation, and although all three cited Siestrunck et al.,[30] none invoked their wave-mechanical stability arguments cited here to analyze off-C-J performance.

In describing the design for an ODW proof-of-principle experiment to be conducted at NASA Ames Research Laboratory, Adelman et al.[49] incorrectly identified the C-J turning angle θ_{cj} of Eq. (32) as the maximum angle of stabilization of an ODW and asserted that Fig. 9 of Ref. 49 showed that the ODW detachment angle θ_{max} was almost 30 deg less than the nonreacting oblique shock detachment angle as the approach Mach number increased from 1 to 10 and (by extrapolation) beyond. However, the present analysis clearly shows that θ_{cj} is the minimum wedge angle for stabilizing an attached ODW and that the difference between the detachment angle θ_{max} for an ODW and for a nonreactive oblique shock in fact approaches zero as the approach Mach number increases, not ca. 30 deg. This issue is of crucial importance with respect to whether variable geometry may be required in the design of ODW engines.

A number of investigators have pursued computational studies of wedge-stabilized combustion phenomena. Bussing and Murman[50] considered lean mixtures of hydrogen/air flowing at $M_1 = 2.5$, $T_1 = 900$ K, past a 10-deg wedge. Using crude ignition delay and exothermic chemical-kinetic assumptions, they predicted shock-induced combustion for fuel/air equivalence ratios of 0.1 and 0.35. The present analysis shows that the leaner condition would admit an overdriven attached ODW, whereas the richer mixture is almost exactly at the C-J condition, corresponding to marginal ODW stabilization. When the equivalence ratio was raised to 0.5, thermal choking was correctly predicted.[50] Cambier et al.[51] considered Mach 4, 800 K flow of stoichiometric hydrogen/air past wedges with turning angles of 18 and 26.5 deg, all well within the stable ODW regime, and predicted overdriven ODWs for all cases

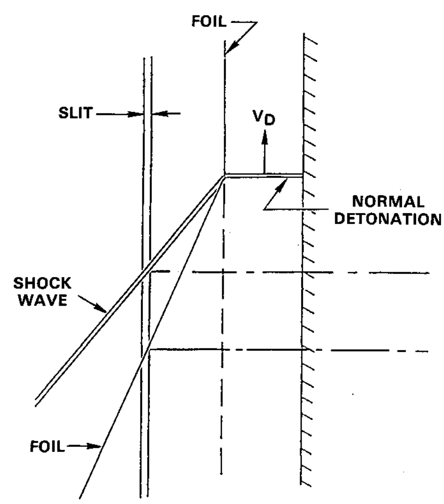

Fig. 12 Schematic of gasdynamic wedge apparatus for producing oblique shock and detonation waves.[56]

considered. Glenn and Pratt[52] considered a Mach 5, 800 K flow of stoichiometric hydrogen/air past a 20-deg wedge, with a kinetic model based on a finite-rate induction time model with instantaneous postinduction heat release,[52,53] and also predicted an overdriven ODW. However, none of the three computational studies referenced[50-52] considered sufficiently detailed chemical-kinetic mechanisms to predict reliably whether shock-induced combustion or an overdriven ODW would occur.

Bogdanoff and Brackett[54] modeled the flow of a fuel-rich mixture of hydrogen/oxygen gas past a ram accelerator projectile/centerbody, as illustrated in Fig. 11. Approach velocities in excess of 6 km/s were required to predict the occurrence of a stable ODW reflected from the outer wall, which is necessary for positive thrust. The forebody cone half-angle of 14 deg is too small to admit stable ODWs for lesser approach velocities, as Fig. 3 graphically illustrates. By increasing the forebody cone half-angle to 17-20 deg, stable operation should be achievable with hydrogen/air mixtures with approach velocities as low as 2-3 km/s[28]—but at the expense of lowering the projectile velocity at which detonation would occur in the bow shock.

Fan et al.[55] and Dabora and Wagner[56] report the experimental stabilization of overdriven ODWs by using a normal detonation wave to drive a gasdynamic wedge through a combustible mixture, as illustrated in Fig. 12.

VI. Summary and Conclusions

The literature on stabilized detonation waves over the past 40 years contains errors of interpretation of experimental results and misconceptions of cause and effect with regard to the interaction of gasdynamic and chemical processes. These errors and misconceptions have inhibited understanding and, therefore, the application of stabilized detonation processes to the propulsion of air-breathing hypersonic aircraft.

For any given fuel-air mixture and freestream temperature, there exists a unique specific heat release Q, and a corresponding M_{cj}, which would result for complete (or equilibrium) combustion. For approach Mach numbers roughly 25% in excess of M_{cj}, there exists a usefully wide range of turning angles between θ_{cj} and θ_{max} within which ODWs may be attached and stabilized. For smaller wedge angles, a stable, complete detonation is not possible and either incomplete detonation, shock-induced deflagration, or no combustion at all may ensue. For wedge angles greater than θ_{max}, the wave will detach and form either a normal detonation or normal shock-induced combustion along the stagnation streamline immediately upstream of the wedge, with the normal wave decaying to a oblique wave off axis. Far from the wall, nonreacting oblique shocks would approach the Mach angle, and oblique detonation waves would either approach oblique Chapmen-Jouguet detonations or would bifurcate into an oblique shock followed downstream by a shock-induced deflagration wave. Detailed kinetic mechanisms or improved global kinetics models, together with the use of highly refined or adaptive gridding, are necessary to predict numerically which conditions might occur in practice.

Stabilized normal or ODWs are supported principally by the mechanical energy of the flowing stream and only secondarily by chemical heat release. Wedges, protrusions, or fuel-air jet interactions are therefore necessary to stabilize either ODWs or shock-induced deflagration waves. Consequently, the Chapman-Jouguet condition has significance only as a limiting design state for stable operation or flameholding. Destructive oscillatory instabilities that may arise in subdetonative flow regions may be avoided by designing to insure that oblique detonations are slightly overdriven, which can be achieved at little cost of total pressure compared to the minimum-total-pressure loss Chapman-Jouguet state.

There exist hypersonic flight regimes where premixing of fuel and air may be desirable or unavoidable due to finite ignition delay times. Consequently, it is essential to understand under what conditions standing detonation waves may occur in order to be able to design supersonic combustors to either avoid the occurrence of oblique detonation waves or to efficiently utilize such waves for propulsion.

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